Problem Set 1

Introductory Linear Algebra

A few matrices and vectors are defined below. Use them to answer the following questions.

$$\mathbf{a} = \begin{bmatrix} 0.2 \\ 2.0 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1.8 \\ 3.5 \\ 8.0 \end{bmatrix}, \mathbf{c} = \begin{bmatrix} 2.1 \\ 5.1 \\ 1.0 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} 5.0 \\ 0.1 \\ 5.4 \\ 2.4 \end{bmatrix}, \mathbf{e} = \begin{bmatrix} 1.0 \\ 2.3 \\ 0.8 \\ 4.0 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 2.0 & 0.1 \\ 0.1 & 3.0 \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 1.0 & 2.0 & 0.4 \\ 2.0 & 6.0 & 1.5 \\ 0.4 & 1.5 & 2.0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0.1 & 1.8 & 3.2 \\ 1.8 & 2.0 & 4.1 \\ 3.2 & 4.1 & 7.0 \end{bmatrix},$$

$$\mathbf{D} = \begin{bmatrix} 8.5 & 4.0 & 2.2 & 0.8 \\ 4.0 & 2.4 & 0.3 & 1.5 \\ 2.2 & 0.3 & 7.3 & 0.1 \\ 0.8 & 1.5 & 0.1 & 7.5 \end{bmatrix}, \mathbf{E} = \begin{bmatrix} 2.0 & 3.8 & 0.2 & 2.2 & 0.1 & 2.6 \\ 3.8 & 0.3 & 2.7 & 3.7 & 3.3 & 0.6 \\ 0.2 & 2.7 & 2.1 & 0.1 & 3.2 & 0.6 \\ 2.2 & 3.7 & 0.1 & 6.9 & 0.8 & 0.7 \\ 0.1 & 3.3 & 3.2 & 0.8 & 4.7 & 1.4 \\ 2.6 & 0.6 & 0.6 & 0.7 & 1.4 & 2.1 \end{bmatrix},$$

$$\mathbf{F} = \begin{bmatrix} 1.0 & 0.4 & 0.5 & 3.2 \\ 0.1 & 2.9 & 1.0 & 8.0 \end{bmatrix}$$

Evaluate the following expressions.

1a. $\mathbf{b}^\dagger\mathbf{c}$
1b. $\mathbf{d}^\dagger\mathbf{e}$
1c. $\mathbf{Bc}$
1d. $\mathbf{Cb}$
1e. $\mathbf{e}^\dagger\mathbf{Dd}$
1f. $(\mathbf{B}+\mathbf{C})\mathbf{b}$
1g. $\mathbf{BC}$
1h. $\mathbf{de}^\dagger$
1i. $\mathbf{Fe}$
1j. $\mathbf{a}^\dagger\mathbf{Fd}$

For the matrices **A**, **B**, **C**, **D**, and **E**, do the following.

2a. Compute the square of the matrix, **M**: $\mathbf{M}^2$
2b. Raise the matrix, **M**, to the third power: $\mathbf{M}^3$
2c. Compute the trace of the matrix.
2d. Evaluate the determinant of the matrix.
2e. Find the eigenvectors and eigenvalues.