Problem Set 2

Postulates of Quantum Mechanics

1. Write down functions (in one dimension) that satisfy the criteria necessary to be valid wavefunctions: single valued, continuous, twice differentiable, and square integrable on the interval $-\infty$ to ∞.

2. Write down functions of x that are eigenfunctions of $\hat{O}$ and their corresponding eigenvalues, where:

   (a) $\hat{O} = \frac{d}{dx}$

   (b) $\hat{O} = \frac{d^2}{dx^2}$

3. Normalize the following functions on the interval $-\infty$ to ∞ (i.e. solve for N such that $\int_{-\infty}^{\infty} f^*(x) f(x) \mathrm{d}x = 1$):

   (a) $f(x) = N\mathrm{e}^{-ax^2}$

   (b) $f(x) = Nx^2\mathrm{e}^{-ax^2}$

   (c) $f(x) = \begin{cases} N\sin\left(\frac{\pi x}{L}\right) & \text{if } 0 \leq x \leq L \\ 0 & \text{otherwise} \end{cases}$

   (d) $f(x) = \begin{cases} N\cos\left(\frac{2\pi x}{L}\right) & \text{if } -L/2 \leq x \leq L/2 \\ 0 & \text{otherwise} \end{cases}$

   Here, a and L are constants.

4. Prove that the functions $\sin(x)$ and $\cos(x)$ are orthogonal on the interval $-\pi$ to π.

Some Potentially Useful Equations

Normalization:	$\int_{-\infty}^{\infty} f^*(x) f(x) \mathrm{d}x = 1$
Orthogonality:	$\int_{-\infty}^{\infty} f^*(x) g(x) \mathrm{d}x = 0$
Eigenvalue problem:	$\hat{O} f(x) = \lambda f(x)$