Problem Set 12

Spin Operators and Spin Functions

1. Evaluate all pairs of commutators between: $\hat{S}_x$, $\hat{S}_y$, $\hat{S}_z$, $\hat{S}_+$, $\hat{S}_-$, and $\hat{S}^2$.

2. Evaluate the action of $\hat{S}^2$ on all possible products of $|\alpha\rangle$ and $|\beta\rangle$ spin functions (for one, two and three electron functions).

3. Construct all unique eigenfunctions of the total spin operator, $\hat{S}^2$, as linear combinations of products of the $|\alpha\rangle$ and $|\beta\rangle$ spin functions (for one, two and three electron functions).

4. Construct matrix representations of the spin operators in a basis of products of spin functions (for one, two and three electron functions).

5. Using a matrix representation of the spin functions and spin operators, repeat part 3.

6. Construct matrix representations of the spin operators in the basis of eigenfunctions of the total spin operator obtained in part 5.

7. Check the commutation relations obtained in part 1 with the commutators of the of the matrix representation of the operators obtained in part 4 (for one, two and three electron functions).

8. For extra practice, repeat parts 2 through 7 for cases with four or more electrons.

Some Potentially Useful Equations

$\langle\alpha|\alpha\rangle = \langle\beta|\beta\rangle = 1$ $\qquad$ $\langle\alpha|\beta\rangle = \langle\beta|\alpha\rangle = 0$

$\hat{S}_x|\alpha\rangle = \frac{\hbar}{2}|\beta\rangle$ $\qquad$ $\hat{S}_x|\beta\rangle = \frac{\hbar}{2}|\alpha\rangle$

$\hat{S}_y|\alpha\rangle = \frac{i\hbar}{2}|\beta\rangle$ $\qquad$ $\hat{S}_y|\beta\rangle = -\frac{i\hbar}{2}|\alpha\rangle$

$\hat{S}_z|\alpha\rangle = \frac{\hbar}{2}|\alpha\rangle$ $\qquad$ $\hat{S}_z|\beta\rangle = -\frac{\hbar}{2}|\beta\rangle$

$\hat{S}_+ = \hat{S}_x + i\hat{S}_y$ $\qquad$ $\hat{S}_- = \hat{S}_x - i\hat{S}_y$

$\hat{S}_+|\alpha\rangle = 0$ $\qquad$ $\hat{S}_+|\beta\rangle = \hbar|\alpha\rangle$

$\hat{S}_-|\alpha\rangle = \hbar|\beta\rangle$ $\qquad$ $\hat{S}_-|\beta\rangle = 0$

$[\hat{S}_x, \hat{S}_y] = i\hbar\hat{S}_z$ $\qquad$ $[\hat{S}_y, \hat{S}_z] = i\hbar\hat{S}_x$

$[\hat{S}_z, \hat{S}_x] = i\hbar\hat{S}_y$

$$\hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 = \hat{S}_+\hat{S}_- - \hbar\hat{S}_z + \hat{S}_z^2 = \hat{S}_-\hat{S}_+ + \hbar\hat{S}_z + \hat{S}_z^2$$